Inter (Part-I) 2019

Mathematics	Group-II	PAPER: I
Time: 2.30 Hours	(SUBJECTIVE TYPE)	Marks: 80

SECTION-I

2. Write short answers to any EIGHT (8) questions: (16)

(i) Prove the rule of addition $\frac{a}{c}+\frac{b}{c}=\frac{a+b}{c}$.

Ans Now, $\frac{a}{c}+\frac{b}{c}$

$$= a.\frac{1}{c}+b.\frac{1}{c}$$

$$=(a+b).\frac{1}{c}$$

$$=\frac{a+b}{c}$$

(ii) Find the multiplicative inverse of $(\sqrt{2},-\sqrt{5})$.

Ans Inverse of $(\sqrt{2},-\sqrt{5})$, $a=\sqrt{2}$, $b=-\sqrt{5}$ is given by

$$\left(\frac{\sqrt{2}}{(\sqrt{2})^2+(-\sqrt{5})^2},\frac{-(-\sqrt{5})}{(\sqrt{2})^2+(-\sqrt{5})^2}\right)=\left(\frac{\sqrt{2}}{2+5},\frac{\sqrt{5}}{2+5}\right)$$

$$=\left(\frac{\sqrt{2}}{7},\frac{\sqrt{5}}{7}\right)$$

(iii) Express the complex number $1+i\sqrt{3}$ in polar form.

Ans Step-I:

Put $r\cos\theta=1$ and $r\sin\theta=\sqrt{3}$

Step-II:

$$r^2=(1)^2+(\sqrt{3})^2$$

$$\Rightarrow \quad r^2=1+3=4 \quad \Rightarrow \quad r=2$$

Step-III:

$$\theta=\tan^{-1}\frac{\sqrt{3}}{1}=\tan^{-1}\sqrt{3}=60^\circ$$

Thus, $1+i\sqrt{3}=2\cos 60^\circ+i\,2\sin 60^\circ$

(iv) Write the power set of {a, {b, c}}.

Ans Let $A=\{a,\{b,c\}\}$

Power set of A is

$$P(A) = \{\phi, \{a\}, \{\{b, c\}\}, \{a, \{b, c\}\}\}$$

(v) Show that the statement $p \to (p \vee q)$ is tautology.

Ans First we will construct truth table for $p \to (p \vee q)$

p	q	$p \vee q$	$p \to (p \vee q)$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	T

Since all the possible values of $p \to (p \vee q)$ are true.

Thus $p \to (p \vee q)$ is a tautology.

(vi) Prove that the identity element e in a group G is unique.

Ans **Theorem:**

If $(G, \dot{\times})$ is a group with e its identity, then e is unique.

Proof:

Suppose the contrary that identity is not unique. And let e′ be another identity.

e, e′ being identities, we have

$e' \dot{\times} e = e \dot{\times} e' = e'$ (e is an identity) (i)

$e' \dot{\times} e = e \dot{\times} e' = e$ (e′ is an identity) (ii)

Comparing (i) and (ii),

$$e' = e$$

Thus the identity of a group is always unique.

(vii) If $A = \begin{bmatrix} 1 & -1 \\ a & b \end{bmatrix}$ and $A^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, find a and b.

Ans $A = \begin{bmatrix} 1 & -1 \\ a & b \end{bmatrix} \Rightarrow A^2 = \begin{bmatrix} 1 & -1 \\ a & b \end{bmatrix}\begin{bmatrix} 1 & -1 \\ a & b \end{bmatrix}$

$$A^2 = \begin{bmatrix} 1(1) + (-1)(a) & 1(-1) + (-1)(b) \\ a(1) + b(a) & a(-1) + b(b) \end{bmatrix} = \begin{bmatrix} 1 - a & -1 - b \\ a + ab & -a + b^2 \end{bmatrix}$$

Now $A^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \therefore \begin{bmatrix} 1 - a & -1 - b \\ a + ab & -a + b^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Comparing corresponding elements, we get

$$1 - a = 1 \qquad \therefore \qquad a = 0$$

and $-1-b=0$ $\therefore$ $b=-1$.

Thus, $\boxed{a=0,\ b=-1}$

(viii) If $B=\begin{bmatrix}5 & -2 & 5\\ 3 & -1 & 4\\ -2 & 1 & -2\end{bmatrix}$, find cofactor B_{21}.

Ans $$B_{21}=(-1)^{1+2}\begin{vmatrix}-2 & 5\\ 1 & -2\end{vmatrix}=-(4-5)=1$$

(ix) **If A is a skew-symmetric matrix, then show that A^2 is a symmetric matrix.**

Ans (i) Let A be symmetric, so $A^t=A$.

$\therefore (A^2)^t=(A.A)^t=A^t.A^t=A.A=A^2$, so A^2 is symmetric.

(ii) Let A be skew-symmetric $\Rightarrow A^t=-A$

$\therefore (A^2)^t=(A.A)^t=A^t.A^t=-A.-A=+A^2$

So, A^2 is skew-symmetric.

(x) **Solve $x^{-2}-10=3x^{-1}$.**

Ans Put $x^{-1}=y$, then the given equation becomes

$$y^2-10=3y$$

$\Rightarrow$ $y^2-3y-10=0$

$\Rightarrow$ $(y-5)(y+2)=0$

$\Rightarrow$ $y=-2, 5$

$\therefore$ $x^{-1}=-2, x^{-1}=5$

$\Rightarrow$ $x=-\frac{1}{2}, x=\frac{1}{5}$

$\Rightarrow$ $S.S=\left\{-\frac{1}{2},\frac{1}{5}\right\}$

(xi) **If α, β are the roots of $x^2-px-p-c=0$, then prove that $(1+\alpha)(1+\beta)=1-c$.**

Ans $$x^2-px-p-c=0$$

Here $a=1, b=?, c=p-c$

$$\alpha+\beta=-\frac{b}{a}=-p$$

$$\alpha\beta=\frac{c}{a}=\frac{p-c}{1}=p-c$$

Now we will prove that

$$(1+\alpha)(1-\beta)=1-c$$

$$L.H.S.=(1+\alpha)(1+\beta)$$

$= 1 + \alpha + \beta + \alpha\beta$
$= 1 + (\alpha + \beta) + \alpha\beta$
$= 1 + p + (-p - c)$
$= 1 + p - p - c = 1 - c = \text{R.H.S.}$

Thus $(1 + \alpha)(1 + \beta) = 1 - c$

(xii) Discuss the nature of roots of the equation $x^2 - 5x + 6 = 0$.

Ans Here, $a = 1, b = -5, c = 6$

Disc. $= D = b^2 - 4ca = 25 - 4(6)(1) = 25 - 24 = 1$

Since (i) $D > 0$, and it is a perfect square, so the roots are rational and unequal.

3. Write short answers to any EIGHT (8) questions: (16)

(i) Define proper fraction.

Ans A rational fraction $\frac{P(x)}{Q(x)}$ is called a Proper Rational Fraction if the degree of the polynomial P(x) in the numerator is less than the degree of the polynomial Q(x) in the denominator. For example, $\frac{3}{x+1}, \frac{2x-5}{x^2+4}$ and $\frac{9x^2}{x^3-1}$ are proper rational fractions or proper fractions.

(ii) If $\frac{x^2 - 10x + 13}{(x-1)(x^2 - 5x + 6)} = \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x-3}$, find value of A.

Ans The factor $x^2 - 5x + 6$ in the denominator can be factorized and its factors are $x - 3$ and $x - 2$.

$$\therefore \quad \frac{x^2 - 10x + 13}{(x-1)(x^2 - 5x + 6)} = \frac{x^2 - 10x + 13}{(x-1)(x-2)(x-3)}$$

Suppose $\frac{x^2 - 10x + 13}{(x-1)(x-2)(x-3)} = \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x-3}$

$\Rightarrow \quad x^2 - 10x + 13 = A(x-2)(x-3) + B(x-1)(x-3) + C(x-1)(x-2)$

which is an identity in x.

Putting $x = 1$ in the identity, we get

$(1)^2 - 10(1) + 13 = A(1-2)(1-3) + B(1-1)(1-3) + C(1-1)(1-2)$

$\Rightarrow \quad 1 - 10 + 13 = A(-1)(-2) + B(0)(-2) + C(0)(-1)$

$4 = 2A$

$\therefore \quad \boxed{A = 2}$

(iii) If $\frac{x}{(x-a)(x-b)(x-c)} = \frac{A}{x-a} + \frac{B}{x-b} + \frac{C}{x-c}$, find value of B.

Ans Let, $\frac{x}{(x-a)(x-b)(x-c)} = \frac{A}{x-a} + \frac{B}{x-b} + \frac{C}{x-c}$ (1)

$\Rightarrow \frac{x}{(x-a)(x-b)(x-c)} = \frac{A(x-b)(x-c) + B(x-a)(x-c) + C(x-a)(x-b)}{(x-a)(x-b)(x-c)}$

$\Rightarrow x = A(x-b)(x-c) + B(x-a)(x-c) + C(x-a)(x-b)$ (2)

Put $x = a$ in eq. (2), we have

$a = A(a-b)(a-c) \Rightarrow \boxed{A = \frac{a}{(a-b)(a-c)}}$

Put $x = b$ in eq. (2), we have

$b = B(b-a)(b-c) \Rightarrow \boxed{B = \frac{b}{(b-a)(b-c)}}$

Put $x = c$ in eq. (2), we have

$c = C(c-a)(c-b) \Rightarrow \boxed{C = \frac{c}{(c-a)(c-b)}}$

Putting the values of A, B and C in eq. (1), we have

$$\frac{x}{(x-a)(x-b)(x-c)} = \frac{a}{(x-a)(a-b)(a-c)} + \frac{b}{(x-b)(b-a)(b-c)} + \frac{c}{(x-c)(c-a)(c-b)}$$

Hence partial fractions are

$$\frac{a}{(a-b)(a-c)(x-a)} + \frac{b}{(b-a)(b-c)(x-b)} + \frac{c}{(c-a)(c-b)(x-c)}$$

(iv) If the numbers $\frac{1}{k}$, $\frac{1}{2k+1}$ and $\frac{1}{4k-1}$ are in harmonic sequence, find k.

Ans $\frac{1}{k}, \frac{1}{2k+1}, \frac{1}{4k-1}$ are in H.P.

$\Rightarrow$ $k, 2k+1, 4k-1$ are in A.P.

$d = (2k+1) - k = (4k-1) - (2k+1)$

$\Rightarrow 2k + 1 - k = 4k - 1 - 2k - 1$

$\Rightarrow k + 1 = 2k - 2$

$\Rightarrow 2k - 2 - k - 1 = 0$

$\Rightarrow k - 3 = 0$

$\Rightarrow \quad k = 3$

(v) Find sum of infinite geometric series 2 + 1 + 0.5 + ---.

Ans Given $\quad 2 + 1 + 0.5 + \ldots$

Here, $\quad a = 2, r = \frac{1}{2}$

Using sum formula for infinite geometric series,

$$S_\infty = \frac{a}{1-r}$$

$$= \frac{2}{1-\frac{1}{2}} = \frac{2}{\frac{1}{2}} = 2 \cdot \frac{2}{1} = 4$$

(vi) Define geometric mean.

Ans A number G is said to be a geometric mean (G.M.) between two numbers a and b, if a, G, b are in G.P. Therefore,

$$\frac{G}{a} = \frac{b}{G} \Rightarrow G^2 = ab \Rightarrow G = \pm\sqrt{ab}$$

(vii) If 5, 8 are two A.Ms between a and b, find a and b.

Ans Given that 5, 8 are two A.M's between a and b.

$\therefore \quad$ a, 5, 8, b are in A.P.

Also, $\quad A_1 = a + d \qquad$ As $\quad d = 5 - a$

$\Rightarrow \quad 5 = a + (5 - a) \quad (1) \qquad$ Also $\quad d = b - 8$ (Difference)

or $\quad 5 = a + (b - 8) \quad (2)$

Subtracting (1) from (2), we get

$a + b - 8 - 5 = 0$

or $\quad a + b - 13 = 0$

or $\quad a + b = 13 \qquad (i)$

and $\quad A_2 = A_1 + d$

$\Rightarrow \quad 8 = 5 + (b - 8)$

or $\quad 8 = b - 3 \quad$ or $\quad b = 8 + 3 \quad$ or $\quad b = 11$

Putting b = 11 in equation (i),

$a + 11 = 13$

$a = 13 - 11$

$a = 2$

Hence $\quad a = 2, b = 11$

(viii) If $\frac{1}{a}, \frac{1}{b}$ and $\frac{1}{c}$ are in A.P, show that $b = \frac{2ac}{a+c}$.

Ans Given $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P.

$$\therefore \quad \frac{1}{b}-\frac{1}{a}=\frac{1}{c}-\frac{1}{b} \qquad \text{(As difference is same in A.P.)}$$

$$\Rightarrow \quad \frac{1}{b}+\frac{1}{b}=\frac{1}{a}+\frac{1}{c}$$

$$\Rightarrow \quad \frac{2}{b}=\frac{c+a}{ac} \qquad \Rightarrow \qquad \frac{1}{b}=\frac{a+c}{2ac}$$

$$\Rightarrow \quad b=\frac{2ac}{a+c}$$

(ix) Prove that $^nC_r = {}^nC_{n-r}$.

Ans If from n different objects, we select r objects, then $(n-r)$ objects are left.

Corresponding to every combination of r objects, there is a combination $(n-r)$ objects and vice versa.

Thus the number of combinations of n objects taken or at a time is equal to number of combinations of n objects taken $(n-r)$ at a time.

$$\therefore \quad {}^nC_r = {}^nC_{n-r}$$

(x) Expand $(1+x)^{-1/3}$ up to 3 terms.

Ans $$(1-2x)^{1/3} = 1+\frac{1}{3}(-2x)+\frac{\frac{1}{3}\left(\frac{1}{3}-1\right)}{2!}(-2x)^2+\ldots\ldots$$

$$= 1-\frac{2}{3}x+\frac{\frac{1}{3}\left(\frac{-2}{3}\right)}{2\,.\,1}(4x^2)+\ldots\ldots$$

$$= 1-\frac{2}{3}x-\frac{4}{9}x^2+\ldots\ldots$$

Putting $x = 0.1$ in the above equation we have

$$(1-2(0.1))^{1/3} = 1-\frac{2}{3}(0.1)-\frac{4}{9}(0.1)^2\ \ldots\ldots$$

$$(1-0.2)^{1/3} = 1-\frac{0.2}{3}-\frac{0.04}{9}\ \ldots\ldots$$

$$(0.8)^{1/3} \approx 1-0.6666-0.00444$$

$$(0.8)^{1/3} \approx 0.9289$$

(xi) Evaluate $\sqrt[3]{30}$ correct to three places of decimal.

Ans $$\sqrt[3]{30} = (30)^{1/3} = (27+3)^{1/3}$$

$$= \left[27\left(1+\frac{3}{27}\right)\right]^{1/3} = (27)^{1/3}\left(1+\frac{1}{9}\right)^{1/3}$$

$$= 3\left(1+\frac{1}{9}\right)^{1/3}$$

$$= 3\left[1+\frac{1}{3}\cdot\frac{1}{9}+\frac{\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)}{2!}\left(\frac{1}{9}\right)^2+\frac{\frac{1}{3}\left(-\frac{2}{3}\right)\left(-\frac{5}{3}\right)}{3!}\left(\frac{1}{9}\right)^3+\ldots\right]$$

$$= 3\left[1+\frac{1}{3}\cdot\frac{1}{9}-\frac{1}{9}\left(\frac{1}{9}\right)^2+\frac{5}{81}\left(\frac{1}{9}\right)^3+\ldots\right]$$

$$= 3\left[1+\frac{1}{27}-\left(\frac{1}{27}\right)^2+\ldots\right]$$

$$= 3[1+.03704-.001372] = 3[1.035668] = 3.107004$$

Thus $\sqrt[3]{30} \approx 3.107$

(xii) Check whether the statement $5^n - 2^n$ is divisible by 3 for $n = 2, 3$ is true or false.

Ans For $n = 2$, we have

$$5^n - 2^n = 5^2 - 2^2 = 25 - 4 = 21$$

It is clearly divisible by 3.

For $n = 3$

$$5^n - 2^n = 5^3 - 2^3 = 125 - 8 = 117$$

which is clearly divisible by 3.

$n = 2, 3$ is true.

4. Write short answers to any NINE (9) questions: (18)

(i) Find r, when $l = 56$ cm, $\theta = 45^\circ$.

Ans $l = 56$ cm, $\theta = 45^\circ \times \frac{\pi}{180} = \frac{\pi}{4} = \frac{22}{7\times 4} = \frac{11}{14}$ radians

$$r = \frac{l}{\theta} = \frac{56}{\frac{11}{14}} = \frac{784}{11} = 71.27 \text{ cm}$$

(ii) Find the values of all trigonometric functions for -15π.

Ans $-15\pi = -(7(2\pi) + \pi)$

The values of the trigonometric functions of the angle -15π are same as the values of the trigonometric functions of the angle π.

$\therefore$ $\sin(-15\pi) = \sin\pi = 0$ $\qquad$ $\cos(-15\pi) = \cos\pi = -1$

$\tan(-15\pi) = \tan\pi =$ undefined. $\cot(-15\pi) = \cot\pi = 0$

$\sec(-15\pi) = \sec\pi = -1$

$\operatorname{cosec}(-15\pi) = \operatorname{cosec}\pi = \text{undefined.}$

(iii) **Prove that $\dfrac{1-\sin\theta}{\cos\theta} = \dfrac{\cos\theta}{1+\sin\theta}$.**

Ans L.H.S $\dfrac{1-\sin\theta}{\cos\theta}$

Multiply and divide by $1+\sin\theta$

$$= \frac{(1-\sin\theta)(1+\sin\theta)}{\cos\theta(1+\sin\theta)}$$

$$= \frac{1-\sin^2\theta}{\cos\theta(1+\sin\theta)} = \frac{\cos^2\theta}{\cos\theta(1+\sin\theta)} \quad \begin{pmatrix}\sin^2\theta+\cos^2\theta = 1 \\ \cos^2\theta = 1-\sin^2\theta\end{pmatrix}$$

$$= \frac{\cos\theta}{(1+\sin\theta)} \quad \text{R.H.S}$$

(iv) **Express the difference $\cos 7\theta - \cos\theta$ as product.**

Ans
$$\cos 7\theta - \cos\theta = -2\sin\frac{7\theta+\theta}{2}\sin\frac{7\theta-\theta}{2}$$
$$= -2\sin 4\theta \sin 3\theta$$

(v) **Prove $\dfrac{1-\cos\alpha}{\sin\alpha} = \tan\dfrac{\alpha}{2}$.**

Ans
$$\text{L.H.S} = \frac{1-\cos\alpha}{\sin\alpha}$$
$$= \frac{2\sin^2\frac{\alpha}{2}}{2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}} = \frac{\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}}$$
$$= \tan\frac{\alpha}{2} = \text{R.H.S.}$$

(vi) **Find the value of $\cos 105^\circ$ without using calculator.**

Ans $\cos 105^\circ = \cos(45^\circ + 60^\circ)$
$$= \cos 45^\circ \cos 60^\circ - \sin 45^\circ \sin 60^\circ$$
$$= \frac{1}{\sqrt{2}}\cdot\frac{1}{2} - \frac{1}{\sqrt{2}}\cdot\frac{\sqrt{3}}{2}$$
$$= \frac{1}{2\sqrt{2}} - \frac{\sqrt{3}}{2\sqrt{2}}$$
$$= \frac{1-\sqrt{3}}{2\sqrt{2}}$$

(vii) **Find the period of $3\sin\frac{2x}{5}$.**

Ans $\because$ $$3\sin\frac{2x}{5} = 3\sin\frac{1}{5}(2x+2\pi)$$
$$= 3\sin\frac{1}{5}(2x+10\pi)$$

Hence period of $3\sin\frac{2x}{5}$ is 10π.

(viii) **With usual notations prove that $\frac{1}{r} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}$.**

Ans $$R.H.S = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}$$
$$= \frac{1}{\frac{\Delta}{s-a}} + \frac{1}{\frac{\Delta}{s-b}} + \frac{1}{\frac{\Delta}{s-c}}$$
$$= \frac{s-a}{\Delta} + \frac{s-b}{\Delta} + \frac{s-c}{\Delta}$$
$$= \frac{1}{\Delta}(s-a+s-b+s-c)$$
$$= \frac{1}{\Delta}[3s-(a+b+c)]$$
$$R.H.S = \frac{1}{\Delta}(3s-2s) \qquad \text{As } s = \frac{a+b+c}{2}.$$
$$= \frac{1}{\Delta}.s = \frac{1}{\frac{\Delta}{s}} = \frac{1}{r} \qquad \text{As } r = \frac{\Delta}{s}$$

(ix) **Define in-circle of the triangle ABC.**

Ans The circle drawn inside a triangle touching its three sides is called its inscribed circle or in-circle. Its centre, known as the in-centre, is the point of intersection of the bisectors of angles of the triangle. Its radius is called in-radius and is denoted by r.

(x) **State the law of tangent. (any two)**

Ans

(i) $$\frac{a-b}{a+b} = \frac{\tan\frac{\alpha-\beta}{2}}{\tan\frac{\alpha+\beta}{2}}$$

(ii) $\dfrac{b-c}{b+c} = \dfrac{\tan\dfrac{\beta-\gamma}{2}}{\tan\dfrac{\beta+\gamma}{2}}$

(xi) Show that $\cos(2\sin^{-1}x) = 1-2x^2$.

Ans L.H.S $= \cos(2\sin^{-1}x)$

Let, $\sin^{-1}x = \theta$

So, $= \cos 2\theta$

$= \cos^2\theta - \sin^2\theta$

$= 1-\sin^2\theta - \sin^2\theta$

$= 1-2\sin^2\theta$

$= 1-2(\sin\theta)^2$

After putting value of θ

$= 1-2\sin^2(\sin^{-1}(x))$

$= 1-2[\sin(\sin^{-1}(x)]^2$

As $\sin[\sin^{-1}(\theta)] = \theta$

$= 1-2(x)^2$

$= 1-2x^2$

= R.H.S

(xii) Solve the equation for $\theta \in [0, \pi]$ $\cot^2\theta = \dfrac{4}{3}$.

Ans $\cot^2\theta = \dfrac{4}{3}$

$\dfrac{1}{\tan^2\theta} = \dfrac{4}{3} \Rightarrow \tan^2\theta = \dfrac{3}{4}$

$\tan\theta = \dfrac{\pm\sqrt{3}}{2}$

$\tan\theta = \dfrac{\sqrt{3}}{2}$, $\qquad \tan\theta = \dfrac{-\sqrt{3}}{2}$

$\theta = \tan^{-1}\dfrac{\sqrt{3}}{2}$ $\qquad \theta = \tan\dfrac{-\sqrt{3}}{2}$

$= 0.7137$ or $40.9^o = -0.7137 = -40.9^o$

Derived fx tan $= \pi n$ where $n \in z$

$\theta = 0.71372$ $\qquad \theta = -0.7173 + n\pi$

$\theta = (40.9^o + \pi n)$ $\qquad \theta = (-40.9 + n\pi)$

S.S $= \{\pm 40.9 + \pi n\}$

(xiii) Solve the equation for $\theta \in [0, \pi]$ $2\sin\theta + \cos^2\theta - 1 = 0$.

Ans

$$2\sin\theta + \cos^2\theta - 1 = 0$$
$$2\sin\theta - (1 - \cos^2\theta) = 0$$
$$2\sin\theta - \sin^2\theta = 0$$
$$\sin\theta\,(2 - \sin\theta) = 0$$

$\therefore$ $\sin\theta = 0$; $2 - \sin\theta = 0$

$\theta = \sin^{-1} 0$; $2 = \sin\theta$

$\theta = 0, \pi$; impossible as $|\sin\theta| \le 1$

Thus, the answer will be $0, \pi$.

SECTION-II

NOTE: Attempt any Three (3) questions.

Q.5.(a) If G is a group under the operation "$\dot{*}$" and $a, b \in G$, find the solutions of the equation: (5)

(i) $a \dot{*} x = b$ (ii) $x \dot{*} a = b$

Ans Given that $a * x = b$

(i) $a \dot{*} x = b$

Pre-multiplying by a^{-1}, we have

$$(a^{-1} \dot{*} a) \dot{*} x = a^{-1} \dot{*} b$$

(by Associative law)

$$e \dot{*} x = a^{-1} \dot{*} b$$
$$x = a^{-1} \dot{*} b$$

which is desired solution.

(ii) $x \dot{*} a = b$

Post-multiplying by a^{-1}, we have

$$(x \dot{*} a) \dot{*} a^{-1} = b \dot{*} a^{-1}$$
$$x \dot{*} (a \dot{*} a^{-1}) = b \dot{*} a^{-1}$$

(by Associative law)

$$x \dot{*} e = b \dot{*} a^{-1}$$
$$x = b \dot{*} a^{-1}$$

which is desired solution.

(b) If 7^{th} and 10^{th} terms of an H.P are $\frac{1}{3}$ and $\frac{5}{21}$, respectively, find its 14^{th} term. (5)

Ans In H.P. $a_7 = \frac{1}{3}$, $a_{10} = \frac{5}{21}$

In A.P. $a_7 = 3$, $a_{10} = \frac{21}{5}$

Thus $a_7 = a + 6d \Rightarrow a + 6d = a$ (i)

and $a_{10} = a + 9d \Rightarrow a + 9d = \frac{21}{5}$ (ii)

(i) – (ii) $\Rightarrow$

$$6d - 9d = 3 - \frac{21}{5}$$

$$\Rightarrow \quad -3d = \frac{15 - 21}{5}$$

$$\Rightarrow \quad -3d = -\frac{6}{5}$$

$$\Rightarrow \quad d = \frac{2}{5}$$

Putting $d = \frac{2}{5}$ in equation (i), we get

$$a + 6\left(\frac{2}{5}\right) = 3$$

$$a + \frac{12}{5} = 3$$

$$a = 3 - \frac{12}{5} = \frac{15 - 12}{5} = \frac{3}{5}$$

Thus, $a = \frac{3}{5}, d = \frac{2}{5}$

Now, $a_{14} = a + 13d$

$$= \frac{3}{5} + 13\left(\frac{2}{5}\right)$$

$$= \frac{3}{5} + \frac{26}{5} = \frac{3 + 26}{5} = \frac{29}{5}$$

$a_{14} = \frac{29}{5}$ in A.P.

$a_{14} = \frac{5}{29}$ in H.P.

Q.6.(a) Show that $\begin{bmatrix} a + l & a & a \\ a & a + l & a \\ a & a & a + l \end{bmatrix} = l^2 (3a + l).$ **(5)**

Ans Adding C_2 and C_3 in C_1, we have

$$\begin{vmatrix} a + l & a & a \\ a & a + l & a \\ a & a & a + l \end{vmatrix} = \begin{vmatrix} 3a + l & a & a \\ 3a + l & a + l & a \\ 3a + l & a & a + l \end{vmatrix}$$

$$= (3a + l)\begin{vmatrix} 1 & a & a \\ 1 & a+l & a \\ 1 & a & a+l \end{vmatrix}$$

$$= (3a + l)\begin{vmatrix} 1 & a & a \\ 0 & l & 0 \\ 0 & 0 & l \end{vmatrix}, \text{ (subt. } R_1 \text{ from } R_2, R_3)$$

(*i.e.*, $R_2 - R_1$, $R_3 - R_1$)

$$= (3a + l)(l^2 - 0) = (3a + l)l^2 = \text{R.H.S.}$$

(b) Prove that ${}^{n-1}C_r + {}^{n-1}C_{r-1} = {}^nC_r$. (5)

Ans L.H.S. $= {}^{n-1}C_r + {}^{n-1}C_{r-1}$

$$= \frac{|n-1}{r\,n-1-r} + \frac{n-1}{|r-1|\,|n-r|}$$

$$= \frac{n-1}{r|r-1|\,|n-r-1|} + \frac{|n-1|}{|r-1|\,(n-r)\,|n-r-1|}$$

$$= \frac{n-1}{r-1\,|n-r-1|}\left[\frac{1}{r} + \frac{1}{n-r}\right] = \frac{|n-1|}{|r-1|\,|n-r-1|}\left[\frac{n-r+r}{r(n-r)}\right]$$

$$= \frac{n|n-1|}{r|r-1|\,(n-r)\,|n-r-1} = \frac{n}{|r|\,|n-r|} = {}^nC_r$$

$$= \text{R.H.S.}$$

Hence ${}^{n-1}C_r + {}^{n-1}C_{r-1} = {}^nC_r$

Q.7.(a) If α, β are the roots of $5x^2 - x - 2 = 0$ form the equation whose roots are $\frac{3}{\alpha}$ and $\frac{3}{\beta}$. (5)

Ans Here $a = 5$, $b = -1$, $c = -2$

If α, β be the roots of $ax^2 + bx + c = 0$, then

$$\alpha + \beta = -\frac{b}{a} = -\frac{-1}{5} = \frac{1}{5}, \qquad \alpha\beta = \frac{c}{a} = \frac{-2}{5} = -\frac{2}{5}$$

$$\text{Sum of roots} = S = \frac{3}{\alpha} + \frac{3}{\beta} = 3\left(\frac{1}{\alpha} + \frac{1}{\beta}\right)$$

$$= \frac{3(\alpha + \beta)}{\alpha\beta} = 3 \cdot \frac{\frac{1}{5}}{-\frac{2}{5}} = -\frac{3}{2}$$

$$\text{Products of roots} = P = \frac{3}{\alpha}\cdot\frac{3}{\beta} = \frac{9}{\alpha\beta} = \frac{9}{\frac{-2}{5}} = -\frac{45}{2}$$

$\therefore$ Required equation is

$$x^2 - Sx + P = 0$$

$$x^2 + \frac{3}{2}x - \frac{45}{2} = 0 \quad \text{or} \quad 2x^2 + 3x - 45 = 0$$

(b) Use mathematical induction to prove that $n! > n^2$ for integral values of $n \geq 4$. (5)

Ans C–1 For $n = 4$

L.H.S $= n! = 4! = 24$

R.H.S $= n^2 = (4)^2 = 16$

Clearly $24 > 16$

Statement is true for $n = 4$

C–2 Suppose the formula is true for $n = k$,

i.e., $k! > k^2$ (i)

C–3 Now we want to prove for $n = k + 1$,

i.e., $(k + 1)! > (k + 1)^2$ (ii)

Multiply by $k + 1$ on both sides of (i), we get

$(k + 1)\, k! > (k + 1)\, k^2$

$\Rightarrow \quad (k + 1)! > (k + 1)(k + 1) \quad \because \; k^2 > k + 1 \; \forall \; k \geq 4$

$\Rightarrow \quad (k + 1)! > (k + 1)^2$

Hence by the principle of mathematical induction, the statement is true for positive integral values of n.

Q.8.(a) A railway train is running on a circular track of radius 500 meters at the rate of 30 km per hour. Through what angle will it turn in 10 sec? (5)

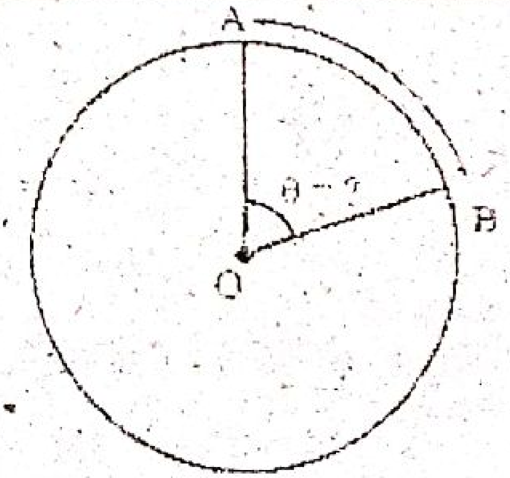
Ans Speed of train = 30 km/h

$$= \frac{30 \times 1000}{60 \times 60} \text{ m/s} = 8.333 \text{ m/s}$$

$\Rightarrow$ In one second, train cover the distance of 8.333 m and in 10 second, train cover the distance

$$= 10 \times 8.333 \text{ m}$$
$$= 83.333 \text{ m}$$

Now consider the Fig.



O = center of circular track

|OA| = |OB| = radius of circular track = γ = 500 m

Now if train start from point A, then in 10 s it cover the distance of 83.333 m (OR you can say that the length of arc AB is 83.333 m) so

Now clearly, we have

$\gamma = 500$ m, $\quad l = 83.33$ m

As $\quad l = r\theta \Rightarrow \theta = \frac{l}{r} = \frac{83.333}{500} = 0.1666 \text{ rad} = \frac{1}{6} \text{ rad.}$

(b) Reduce $\sin^4\theta$ to an expression involving only function of multiples of θ raised to the first power. (5)

Ans

$$\sin^2\theta = \frac{1-\cos 2\theta}{2}$$

$$(\sin^2\theta)^2 = \left(\frac{1-\cos 2\theta}{2}\right)^2$$

$$\sin^4\theta = \frac{1+\cos^2 2\theta - 2\cos 2\theta}{4}$$

$$= \frac{1}{4}\left\{1 + \left(\frac{1+\cos 4\theta}{2}\right) - 2\cos 2\theta\right\}$$

$$= \frac{1}{4}\left\{\frac{2+1+\cos 4\theta - 4\cos 2\theta}{2}\right\}$$

$$= \frac{3+\cos 4\theta - 4\cos 2\theta}{8}$$

$$\boxed{\text{Hence } \sin^4\theta = \frac{3-4\cos 2\theta + \cos 4\theta}{8}}$$

Q.9.(a) Prove that $r_1r_2 + r_2r_3 + r_3r_1 = s^2$. (5)

Ans L.H.S. $= r_1r_2 + r_2r_3 + r_3r_1$

$$= \frac{\Delta}{s-a}\cdot\frac{\Delta}{s-b} + \frac{\Delta}{s-b}\cdot\frac{\Delta}{s-c} + \frac{\Delta}{s-c}\cdot\frac{\Delta}{s-a}$$

$$= \frac{\Delta^2}{(s-a)(s-b)} + \frac{\Delta^2}{(s-b)(s-c)} + \frac{\Delta^2}{(s-c)(s-a)}$$

$$= \Delta^2 \left[\frac{1}{(s-a)(s-b)} + \frac{1}{(s-b)(s-c)} + \frac{1}{(s-c)(s-a)}\right]$$

$$= \Delta^2\left(\frac{s-c+s-a+s-b}{(s-a)(s-b)(s-c)}\right)$$

$$= \frac{\Delta^2\,[3s-(a+b+c)]}{s(s-a)(s-b)(s-c)} \times \frac{s}{1}$$

$$= \frac{\Delta^2(3s-2s)}{\Delta^2} \times s \qquad \text{As} \quad s = \frac{a+b+c}{2}$$

$$\Rightarrow \quad 2s = a+b+c$$

$$= \frac{s}{1} \times s = s^2 = \text{R.H.S}$$

(b) Prove that $\tan^{-1} A + \tan^{-1} B = \tan^{-1} \frac{A+B}{1-AB}$. (5)

Ans Let $\tan^{-1} A = x \quad \Rightarrow \quad \tan x = A$

and $\tan^{-1} B = y \quad \Rightarrow \quad \tan y = B$

Now, $\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y} = \frac{A+B}{1-AB}$

$$\Rightarrow \quad x + y = \tan^{-1} \frac{A+B}{1-AB}$$

$$\therefore \quad \boxed{\tan^{-1} A + \tan^{-1} B = \tan^{-1} \frac{A+B}{1-AB}}$$